Fact Mastery and More!

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Fact Games

Overview for Teaching Math Facts

Some Issues in Teaching Facts

All too often we hear teachers lament that their students don't know their math facts. In class, students guess, freeze up, count with their fingers, tap their pencils, or simply utter the wrong answer. Adding to this concern are questions about the role of facts in "new" or reform mathematics. Do students really need to know their math facts? Can't they just use times tables or calculators? Only after a sincere effort to teach facts strategically and systematically do we feel that the answers to those questions might be yes. Instead, it is our first obligation to get students to *know* facts. As you will see with the rest of our materials, the purpose for having a ready knowledge of facts has more to do with mental computations and approximation skills than it does with working computation problems by hand. Having an ability to manipulate numbers in your head and on paper in a flexible manner is still critical to mathematical thinking. It also facilitates the ability to do any higher mathematics. For example, a simple algebra problem such as $4x + 1 = 25$ is much harder for students who don't know their facts. The lack of fact knowledge gets in the way of a mastery of procedures and conceptual understanding.

Undoubtedly, there are many reasons why children don't know their facts. One common reason is that we don't provide enough practice. Facts are often taught "all at once," and some children are overwhelmed by the quantity of material that needs to be memorized. As a consequence, some facts are partially learned and some are never learned. In his book, *The Number Sense,* Stanislas Dehaene argues that many students are also likely to confuse their facts because of the way this information is stored in associative memory. It is easy to skip from one fact to another one and produce the wrong answer (e.g., mentally skip from 8 x 7 to 9 x 6 and think that 8 x 7 = 54). A host of researchers who specialize in the area of at-risk students and children with learning disabilities suggest that these students may simply take a longer period of time to develop competence or "automaticity" in math facts than average ability students. None of this, however, should prevent us from doing the best job we can to teach facts. That implies sound principles for teaching facts and a *consistent* scheme for teaching facts over time.

The Purpose of These Materials

The fundamental purpose of these materials is to develop fluency in math facts. We define fluency as knowing the answer to a fact in three seconds or less. Optimally, students should know facts in a little over one second. What you will find, however, is a significant departure from the time-honored tradition of giving students stacks of flash cards to memorize.

The facts materials that follow are guided by four main principles. First, it is important, where possible, to give students **strategies** that will facilitate learning facts. Rather than treating each addition fact, for example, as an isolated piece of knowledge, many students can benefit from a strategic approach to math facts. Useful strategies for teaching facts can be found throughout our math facts materials. In fact, *you will notice that at times,*

more than one strategy can apply to some facts. We have done this to stress the role of strategic understanding in learning math facts.

Second, we try to link strategic understanding with **visual representations**. For example, we believe that it is highly useful to link simple addition facts to the use of the number line. A tool called "ten frames" is a useful way of teaching addition and subtraction facts. Arrays, fact family cards, and number lines are good visual representations for multiplication and division. We will describe how these representations are used in the different units on the basic fact operations. We have also provided templates for each of these different representations in a separate section.

Third, practice to mastery requires a **careful orchestration of new and review facts**. That is, teaching facts can also be viewed as the kind of learning required to master the vocabulary of a foreign language or technical terms in a science course. Each set of facts should have a careful mix of new facts (generally, between 3 to 7) and a systematic review of previously taught facts. Educational and psychological research indicates that providing extensive or "massed" practice of new facts and systematic review or "distributed" practice on previously taught facts is one of the most efficient ways to achieve mastery. Put simply, this structure helps students achieve fluency. The time limits for completing our fact practice pages also help develop fluency.

Finally, it is important for students to see how basic facts generalize. That is, it is *critical* that students see the link between 3 + 2, 30 + 20, and 300 + 200. These are called extended facts, and the purpose of these exercises is to develop a wider sense of numbers. This knowledge is also crucial to mental computation and approximation skills.

What's Here

Structure of the Facts Units

The following materials are grouped according to the four basic operations: addition, subtraction, multiplication, and division. And here's something that's very important for efficiency purposes. Each operation is divided into two "units" or clusters of materials. For example, addition is comprised of "easy addition" and "hard addition" units. The same structure holds true for subtraction and multiplication. As you will also see, division facts have their own practice structure. One purpose for grouping facts this way is so that students can have more time to practice learning the harder facts. In other words, there's no need to provide extensive or "massed" practice on facts that students already know. Consequently, the hard facts units are just that – they provide extensive practice on facts that are difficult to learn. There is also a random review of previously taught facts.

You will find the following materials in *Just the Facts*.

- *Pretests* for Placing Students
- *Templates* to Use for Visually Representing Strategies
- *Easy and Hard Units* of Facts for Each Operation
    - Overview of the strategies and suggestions for teaching
    - Daily sets of facts
        - Massed practice on facts that follow a specific strategy
        - A random review of previously taught facts
        - Extended facts

Pretests

In order to determine where to place your students, there is a pretest for each unit. If students can demonstrate mastery on the pretest (complete the test in two minutes or less with 90 percent accuracy), then they can move on to the next unit. We strongly recommend a high level of mastery – 90 percent correct and 3 seconds per fact. Otherwise, students tend to continue counting on their fingers, in their head, or tap their pencils to get answers.

Templates for Visually Representing Facts

There are also templates for visual representations that can be used with different types of facts. Number lines and ten frames work well with addition and subtraction. Fact family triangles, arrays, and number lines can be used with multiplication and division.

Easy and Hard Units of Facts for Each Operation

Each unit begins with a list of strategies that apply to the following *sets* of facts. After each strategy overview, you will find three types of practice sheets.

Cover Sheet: Strategy practice page. This sheet has new facts in a box at the top of the page. Students should be given an open-ended amount of time to complete this sheet so that they can thoughtfully apply the strategies.

Practice Sheets: Timed practice pages. As mentioned, these facts are a systematic mix of new and old facts. Each sheet is timed. Mastery criterion is 90% correct (36 facts) in 2 minutes or less.

Extended Facts Sheet: These facts foster a generalized understanding of facts. You do not have to time these facts. Instead, it is important to discuss how these practice problems relate to each other.

Final Notes

First, we do not know your classroom as well as you do. You will probably need to create more practice sheets based on student performance. It is easiest to just photocopy the same practice pages and have students complete them again. You can also create your own sheets. Identify which facts are causing the greatest trouble and make sure that there are even more of them on the page. Also, you may want to quickly review those facts at other times of the day or assign them as part of the homework.

Second, make sure that you practice on a *regular basis*. Optimally, they should do it every day. We believe that you can provide daily fact practice in less than 7 minutes. That includes having students swap papers and grade them as a group. One of the key reasons that so many students do not know their facts is a lack of *systematic* practice. Therefore, make brief, structured math fact practice a part of your daily class.

Getting Started

1. **Pretest your students. If they do not meet criterion on a unit (below 90% and/or more than 2 minutes to complete the page), then place them in the unit. If they are close to criterion, you may want to think about just teaching the few facts that they do not know.**

2. **Begin teaching the unit. For example, if they know addition and easy subtraction facts, but they do not meet criterion on hard subtraction, start teaching hard subtraction. Look at the strategy pages before you teach a set of facts.**

Math Fact Pretests

You can find the following pretests by their codes. The codes for each test are just above the name on the page. Remember, the pretests are timed tests. We strongly recommend 2 minutes for the test (3 seconds per item). To meet criterion on the test, the student should complete 36 correct.

*There are no division pretests because we treat division as multiplication.

Name_____ Date_____

1 + 6	4 + 4	0 + 3	8 + 2	7 + 2
4 + 2	8 + 8	2 + 5	1 + 4	7 + 1
2 + 8	8 + 1	2 + 3	6 + 1	3 + 3
1 + 3	0 + 7	4 + 1	6 + 6	6 + 2
9 + 1	5 + 5	2 + 1	1 + 8	8 + 0
1 + 2	1 + 5	2 + 2	9 + 9	5 + 2
7 + 7	2 + 7	1 + 1	6 + 0	3 + 1
5 + 1	3 + 2	2 + 6	7 + 1	2 + 4

Name_____ Date_____

3 **+ 7**	**4** **+ 8**	**7** **+ 9**	**6** **+ 5**	**5** **+ 8**
9 **+ 6**	**4** **+ 7**	**3** **+ 9**	**5** **+ 7**	**6** **+ 3**
8 **+ 5**	**3** **+ 5**	**7** **+ 3**	**9** **+ 2**	**8** **+ 7**
6 **+ 4**	**8** **+ 3**	**9** **+ 5**	**6** **+ 7**	**7** **+ 5**
4 **+ 3**	**6** **+ 8**	**5** **+ 9**	**4** **+ 6**	**8** **+ 4**
2 **+ 9**	**5** **+ 3**	**8** **+ 9**	**3** **+ 6**	**7** **+ 4**
9 **+ 4**	**3** **+ 8**	**9** **+ 3**	**7** **+ 8**	**6** **+ 9**
9 **+ 8**	**7** **+ 6**	**9** **+ 7**	**3** **+ 4**	**8** **+ 6**

Name_____ Date_____

8 - 0	4 - 2	3 - 3	7 - 5	6 - 1
5 - 3	10 - 8	9 - 7	8 - 8	5 - 2
3 - 2	8 - 2	5 - 0	10 - 2	6 - 3
4 - 1	8 - 4	9 - 9	4 - 3	8 - 6
8 - 7	6 - 2	9 - 1	5 - 5	7 - 1
10 - 10	9 - 2	3 - 1	2 - 2	8 - 1
2 - 1	7 - 6	6 - 5	7 - 2	10 - 9
6 - 6	6 - 4	10 - 1	5 - 4	9 - 8

Name_____ Date_____

13 **- 8**	**11** **- 9**	**7** **- 4**	**9** **- 5**	**13** **- 7**
13 **- 5**	**7** **- 3**	**11** **- 6**	**8** **- 3**	**12** **- 6**
9 **- 4**	**13** **- 6**	**10** **- 4**	**14** **- 8**	**9** **- 6**
14 **- 5**	**16** **- 9**	**13** **- 9**	**15** **- 6**	**10** **- 7**
12 **- 7**	**15** **- 8**	**14** **- 6**	**12** **- 8**	**15** **- 9**
11 **- 8**	**12** **- 3**	**13** **- 4**	**16** **- 8**	**8** **- 5**
17 **-9**	**10** **- 3**	**12** **- 9**	**15** **- 7**	**14** **- 9**
11 **- 7**	**17** **- 8**	**12** **- 5**	**10** **- 6**	**16** **- 7**

Name_____ Date_____

9 x 1	10 x 3	7 x 1	0 x 4	10 x 7
5 x 7	9 x 5	6 x 1	3 x 5	6 x 10
10 x 6	1 x 4	5 x 8	2 x 2	4 x 1
5 x 9	10 x 4	10 x 0	8 x 5	10 x 2
3 x 1	10 x 10	7 x 5	5 x 3	2 x 5
10 x 8	2 x 3	5 x 5	1 x 8	10 x 9
2 x 4	9 x 2	2 x 10	1 x 6	2 x 7
5 x 4	7 x 10	6 x 5	10 x 5	8 x 2

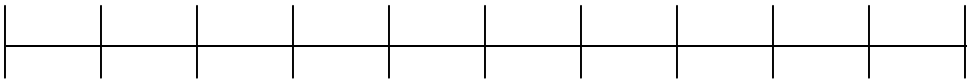
Name_____ Date_____

4 x 9	6 x 7	3 x 7	6 x 9	7 x 8
7 x 4	3 x 9	5 x 5	8 x 3	4 x 8
8 x 8	6 x 4	8 x 7	7 x 3	6 x 7
3 x 3	6 x 8	6 x 3	7 x 9	9 x 3
3 x 4	9 x 6	8 x 6	3 x 8	4 x 4
7 x 6	4 x 6	4 x 7	9 x 7	8 x 4
4 x 9	8 x 9	6 x 6	4 x 3	8 x 8
5 x 4	9 x 9	3 x 6	7 x 7	9 x 8

Templates for Representing Math Facts

Ten Frames

Number Line



Fact Family Triangles