SUPPORTING STUDENTS IN MATHEMATICS THROUGH THE USE OF MANIPULATIVES

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A society can claim success in eradicating the malady of mathematics illiteracy if and only if all its progeny are able to develop to their fullest potential. If its offspring can become employable workers, wisely choosing consumers, and autonomously thinking citizens who can be contributors in the super symbolic quantitative world they will inherit, then society can say, "Victory is ours!" Elliott and Garnett (1994, p.15)

Introduction

Is it possible for *all* children to become mathematically literate? To achieve this goal of mathematics literacy and to meet the needs of all children requires a change in our thinking about the framework of mathematics curricula and how children learn mathematics. The National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* in 1989 and its revised framework of *Principles and Standards for School Mathematics* in 2000 provides a vision for *all* students to think mathematically and highlights learning by *all* students.

The purpose of this brief is to: 1) discuss educational reforms that have implications for mathematics instruction and students with learning disabilities; 2) discuss the NCTM standards and principles and why the "process" standards provide a foundation for more effective learning of mathematics; 3) show how implementation of the process standards creates access to the general education curriculum; 4) provide an example of a research-based instructional intervention that supports the attainment of the goals of mathematical literacy for all students; and 5) suggest some interactive, online simulations (virtual manipulatives) and other technological resources that can build conceptual knowledge in math for all students, specifically for students with disabilities. This brief is designed to provide information to state and district special education administrators and technical assistance providers.

Mathematics Literacy and Children with Disabilities

Mathematics literacy is the ability to apply skills and concepts, reason through, communicate about, and solve mathematical problems (NCTM, 1989). Mathematics instruction involves the pedagogical strategies, curricular materials, and assessments that help all students master the skills and concepts relevant to the development of mathematical literacy. From the earliest grades and throughout their school experiences, children should feel the importance of success in solving problems, figuring things out, and making sense of mathematics. In fact, raising expectations for mathematical reasoning, communicating, making connections, using representations, and problem solving has led to higher standards of performance in mathematics. This requires that students acquire and retain a broad range of mathematical skills and concepts and processes to learn the mathematics curriculum.

What does this mean for students with disabilities? Although every student is affected by the increasing demands and expectations in mathematics, students with disabilities are placed at an even greater disadvantage because of the difficulties they tend to experience in acquiring and retaining knowledge (Miller and Mercer, 1997). Many students with mild disabilities experience difficulty with mathematics due to characteristics that impede their performance, especially in problem solving and computation (Maccini & Gagnon, 2000). Deficits in mathematics performance may be as serious a problem for these students as the reading deficits commonly attributed to characteristics of learning disabled (LD) students (Mastropieri, Scruggs, Shiah, 1991).

Several research studies have described students with LD who exhibited deficits in both mathematics computation and problem solving (Cawley, Miller, & School, 1987; Englert, Culatta, & Horn, 1987; Scruggs & Masterpieri, 1986), as well as the execution of specific mathematics strategies (Swanson & Rhine, 1985). Cawley and Miller (1989) reported that eight- and nine-year-olds identified as LD performed at about a first-grade level on computation and application. Fleischner, Garnett, and Shepherd (1982) found that sixth-graders with LD solved basic addition facts no better than third-graders without disabilities.

Cawley, Parmar, Yan, and Miller (1996) found in their research studies that while typically mainstream students learn mathematical concepts at a steadily increasing pace, students with learning disabilities acquire skills in a broken sequence and have lower retention rates than their non-disabled peers. These retention problems increase as the concepts become more difficult. Specifically, Miles and Forcht (1995) reported that many students with LD demonstrated problems when they first encountered algebraic concepts because of the symbolic or abstract reasoning involved.

Baroody and Hume (1991) remind us that most children with LD are not intellectually impaired but require instruction that is developmentally appropriate to the ways children think and learn. Instruction should focus on: 1) understanding; 2) learning that is active and purposeful; 3) linking formal instruction to informal knowledge; and 4) encouraging reflection and discussion. More specifically, mathematics instruction for all children, including those with LD, should: 1) promote a broad range of mathematical concepts that go beyond computation and include geometry and fractions; 2) actively involve students in doing mathematics that have a purpose; 3) encourage and build on children's strengths and their informal knowledge; and 4) encourage students to justify, discuss, and compare ideas and strategies.

How the Process Standards Promote Access for Students with LD

All students need to have the ability to solve problems, make connections within mathematics and with other disciplines, and represent mathematics in different forms visually and abstractly. The NCTM process standards encourage instruction in which students access mathematics through an understanding of mathematical concepts. For students with LD, the process standards become even more important to the development of the process skills within the strategies designed to assist LD students in bridging the gap between "doing" mathematics and "knowing" and understanding the mathematics curriculum.

The NCTM content and process standards (http://www.nctm.org), along with the *Professional Standards for Teaching Mathematics,* provide the supporting framework and the strategies for all teachers, including teachers of students with special needs, to give every student an opportunity to be successful in mathematics. This framework for mathematics instruction encourages all students to have numerous and varied experiences that allow them to solve complex problems, read, write, and discuss mathematics, test and build arguments about a conjecture's validity, and to value mathematics as a connection with the real world (NCTM, 1988). A set of overarching principles within the NCTM framework and research-based strategies are a means of helping all students become more successful and confident mathematical thinkers.

One of the principles in the NCTM framework is the equity principle that opens the door for all students to engage in mathematical content and processes. "All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics." (NCTM, 2000) Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access to, and attainment of, mathematics for students with disabilities. The equity principle does not provide a prescribed way of achieving success in mathematics for students with disabilities, but it promotes an approach (method) through the process standards as a foundation to build upon the understanding of mathematical content in accessing the general mathematics curriculum and encourages students to become independent learners and thinkers of mathematics.

Given the diversity of students, it is unrealistic to assume that one curriculum or set of standards will suffice to meet the mathematical needs of every student. Carnine (1992) suggests that one method to teach mathematics to all students is not likely, especially considering the special needs of students with learning problems. Most individuals with LD need accommodations or modifications in texts, materials, assignments, teaching methods, tests, and homework (Bateman, 1992). Students need individualization to address the specific mathematical disability that emerges from the unique learning characteristics of the student. The process standards make learning mathematics accessible to students with disabilities. For students to effectively engage in and understand mathematical content and processes and to align all students with the equity principle recommended by NCTM, we must integrate these standards and principles with effective instructional interventions for students with disabilities. The remainder of this brief discusses one research-based instructional approach to teach mathematics to children with LD.

Concrete-Representational-Abstract Instructional Approach

Children with LD often have difficulty with symbolic or abstract concepts and reasoning. These students may need extra assistance through hands-on manipulatives and pictorial representations of mathematical concepts. Hands-on experiences allow students to understand how numerical symbols and abstract equations operate at a concrete level, making the information more accessible to all students (Devlin, 2000; Maccini & Gagnon, 2000).

One effective intervention for mathematics instruction that research suggests can enhance the mathematics performance of students with LD is the concrete-representational-abstract (CRA) sequence of instruction. CRA is a three-part instructional strategy with each part building on the previous instruction to promote student learning and retention, and addresses conceptual knowledge of students with LD. The CRA sequence of instruction incorporates the use of hands-on manipulatives in the concrete stage, followed by pictorial displays in the representations phase, and in the next phase facilitates abstract reasoning with numerical symbols. Learning disabled students learning basic mathematics facts with CRA instruction show improvements in acquisition and retention of mathematical concepts (Miller & Mercer, 1993). CRA supports understanding of underlying mathematical concepts before learning "rules," that is, moving from a concrete model of chips or blocks to an abstract representation (4 x 3 = 12). According to VanDeWalle (2001, p. 425) conceptual understanding is essential to mathematics proficiency. "Do not be content with right answers. Always demand explanations." The effectiveness of CRA and the strategy is described below.

Research on Effectiveness of CRA

CRA provides a strategy for students to gain an understanding of the mathematics concepts/skills they are learning. Teaching mathematics through a CRA sequence of instruction has abundant support for its effectiveness for students with LD (Harris, Miller and Mercer, 1995; Mercer, Jordan, & Miller, 1996; Mercer & Mercer, 1993; Mercer & Mercer, 1998; Peterson, Mercer, & O'Shea, 1988) and for students without learning disabilities (Baroody, 1987; Kennedy & Tipps; 1998; VanDeWalle, 1994). When students with LD are allowed to first develop a concrete understanding of the mathematics concept/skill, they are more likely to perform that mathematics skill and develop the conceptual understanding of the mathematics concept at the abstract level.

Research-based studies show that students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations (Harrison & Harrison, 1986; Suydam & Higgins, 1977). Structured concrete materials have been used as a foundation to develop concepts and to clarify early number relations, place value, computation, fractions, decimals, measurement, geometry, money, percentage, number bases, story problems, probability, and statistics (Bruni & Silverman, 1986). A description of CRA follows. Each stage in the sequence includes examples and teacher guidelines.

Strategy Description

The CRA instructional sequence consists of three stages; the concrete, the representational, and the abstract, and promotes understanding of mathematical concepts for students with LD. A sample problem is used below to illustrate these stages.

<u>Sample Problem</u>: Multiplication (Repeated Addition)

<u>Objective</u>: Student models multiplication problem as repeated addition of 3 groups of 4 using chips, then drawing a model, then converting to the abstract mathematical language of numbers.

Rachel gives 4 cookies to each of her 3 friends. How many cookies does she give out altogether?

- *Concrete.* In the concrete stage, instruction proceeds through a sequence with each mathematical concept first modeled with concrete materials, i.e., red and yellow chips, cubes, base-ten blocks, pattern blocks, fraction bars, etc. In the figure below, the student uses chips to represent cookies in the problem, 3 groups of 4 cookies (see Figure 1). These materials by themselves are not enough. The concrete model must work together with teacher guidance, student interactions, repeated teacher demonstrations and explanations, and many opportunities for students to practice and demonstrate mastery of concepts. Suggested materials and prompts are included in the teacher guidelines.

- *Representational.* In the representational stage, the mathematics concept is modeled at the semi-concrete level which may involve drawing pictures that represent concrete objects (e.g., circles, dots, tallies, stamps imprinting pictures for counting). For the sample problem above, the student uses circles to indicate cookies, and associates each group/friend with a box/oval around the cookies (see Figure 2). Again, students are provided many opportunities for practice and to demonstrate mastery of the mathematics concept.

- *Abstract.* In this stage, the mathematics concept is modeled at the abstract level using only numbers, notation and mathematical symbols (see Figure 3). The student writes a numerical representation of the cookies to find the total (4 + 4 + 4 = 12) through repeated addition or (4 x 3 = 12) through multiplication. Multiple opportunities for practice and demonstration should be provided to achieve mastery of the mathematics concept.

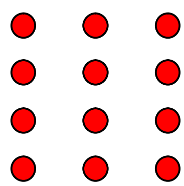
Example (Figures 1-3): Concrete – Representational – Abstract Instructional Sequence

The figures below describe what the student is doing and what the teacher is doing at each of these levels.

Problem: Rachel gives 4 cookies to each of her 3 friends. How many cookies does she give out altogether?

Figure 1

Concrete Stage:



Teacher Guidelines:

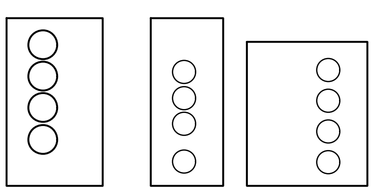
Provide 16-20 chips (manipulatives) in front of the student. Ask the student to use the chips to show what the problem represents. How many cookies? How many friends? (Use more chips than needed so the student decides on the number.) Some students may need a board, a specific area to arrange the chips. Some students may arrange the chips in columns or in one row or in clusters of 4 each. Alternative representations are correct as long as the student shows 3 groups of 4. If the student's response is correct, reinforce the student positively. If incorrect, have the student touch each chip and count to 4, repeating until the student sees 3 groups of 4. Push remaining chips aside.

Student:

Uses chips to model the total number of cookies given to friends. Student can touch the chips and count all the chips aloud or teacher can prompt the student to count alone.

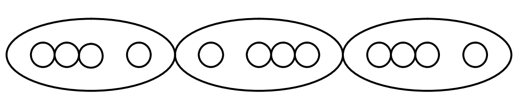
Figure 2

Representational Stage:



Teacher Guidelines:

Provide a paper and pencil, crayon or chalk for the student to draw a model showing the number of cookies each friend has. Circles do not have to be perfect circles. These are pictures of cookies representing a number for each friend. Suggest that the student draw a circle or box around the groups of 4. The teacher can provide a string or yarn to help students group these pictures of four. Numbers are important. Student circles may not be perfect in size or shape.



Student:

Draws pictures of 4 small circles representing cookies in groups of 3. The student draws a box , a circle, or a figure around each group of 4 circles or cookies.

Figure 3

Abstract Stage:

$\square \quad \square \quad \square \quad\quad \square$

$4 \quad + \quad 4 \quad + \quad 4 \quad = \quad 12$

$3 \times 4 = 12$

Teacher Guidelines:
Prompt the student to point with his/her finger or touch the chips in each group. What is the number of chips in each group? Have the student write the number inside a box. Repeat until the student identifies all 3 groups. “How many is that altogether?” “Write that number in the last box.” A student may need more prompting. Write plus signs between the boxes and explain that counting all the chips in 3 groups of 4 is the same as adding 4 + 4 + 4. The total is equal to 12. Is this a reasonable answer? Help the students make a connection that 3 groups of 4 is the same as saying 3 x 4. Ask the students to explain, “How is repeated addition the same as multiplying 3 times 4?”

Student:
Counts and writes the numerical representation of the groups and the total number. The student can explain how he/she arrived at this conclusion or answer.

Extension: Ask the students to model a similar problem. Repeat the steps for 2 friends and 3 cookies.

The CRA sequence provides a graduated and conceptually supported framework for students to create a meaningful connection between concrete, representational, and abstract levels of understanding. Beginning with visual, tactile, and kinesthetic experiences to establish their understanding of numbers, students expand their understanding through pictorial representations of the concrete objects and move to the abstract level of understanding the meaning of numbers (number sense). Teachers can prompt students with questions at each stage as indicated in Figure 4. Teachers can also read the problem aloud and summarize what the student completed as the students moves sequentially through the stages using models, verbalization, drawings, and numerical representations to indicate each step in order. When implementing this strategy, teachers recognize good instruction by referring to concepts or activities in the different stages. For reinforcement of concepts, instruction may be cyclic, not just a linear sequence of instructional tasks.

Figure 4	**Prompts or Questions for Students**
Concrete:	How did you model this? What did you show?
Representational:	Could you draw a model of this? How did you do that? What did you draw to show groups? How did you group these?
Abstract:	What numbers and operations did you use to show this problem? Is this answer a reasonable number?

Students can also achieve a better understanding of the mathematical content of multiplication of numbers and number sense by creating new problems and practicing the CRA instructional sequence through the concrete, representational, and abstract stages (numerical symbols and numbers).

Although some students may not need to draw pictures to make connections with the abstract concept, students gain confidence and reinforce the concrete understanding by making drawings similar to the manipulative, and thus become more independent problem solvers. Multiple experiences with problems like this allow students to internalize the problem-solving process and give them the capability of duplicating the process.

Thus, the CRA instructional sequence becomes a valuable intervention for students with LD to learn the NCTM process standards of problem solving, reasoning and proof, communications, connections, and representations. CRA also provides a process for problem solving applicable to every age group, in informal and formal learning situations for students with LD. It establishes background knowledge, and makes students confident with an approach to reason and make connections for more complex problem solving situations.

Figure 5 shows how CRA is closely matched to the NCTM process standards. In this alignment, the CRA strategy connects the way students learn mathematics with the way students “do” and “know” mathematics through the NCTM process standards. The process standards cut across the NCTM content standards to allow accessibility to the general mathematics curriculum for *all* students.

Figure 5

NCTM Process Standards	CRA Instructional Sequence (Sample Problem)
Problem-Solving Students	**Concrete Stage**
• “do” mathematics to build knowledge	- represent numbers from word problem
• develop strategies for problem solving	- initiate CRA strategy
• build new ideas	- arrange groups of 4, repeated addition
Representations Students	**Representational Stage**
• express math number as a circle or tally	- draw a circle to model a number of chips
• express math idea as a box or oval	- model a group of 4 with an oval or box
• understands language, symbol, and notation	- represent language (4), symbol (+ chips)
Communications Students	**Concrete/Representational/Abstract**
• talk about what they did with the mathematical idea	- use the number of chips needed (C)
• describes how they arranged the chips in groups and why	- indicate groups with circles (R)
• explains how they arrived at the answer	- count using repeated addition (A)
Reasoning and Proof Students	**Concrete/Representational/Abstract**
• find patterns of 4s	- reasoned that 3 groups of 4 chips (C, R)
• linked 3 groups of 4 to find the answer	- find the total with repeated addition (A)
• investigated connection of addition & multiplication	- linked repeated addition to multiplication (A)
Connections Students	**Abstract**
• apply repeated addition to solve real-world problems	- understand multiplication is repeated addition
• connect abstract numbers to concrete models	- 4 x 3 =12
• connect geometric figures to numbers	- draw with ovals and squares to make groups

In today’s challenging mathematics classroom composed of students with diverse backgrounds and abilities, teachers seek strategies and activities to assist student’s learning and understanding of mathematics. Quality professional development combined with proven educational technology as part of the mathematics curriculum may assist educators in achieving this goal. Virtual manipulatives and graphing calculators replace the concrete manipulatives to provide visualization, help students make connections, and understand mathematical relations at the touch of a button.

As an example, Texas Instruments offers several varieties of graphing calculators designed to demonstrate and manipulate geometric objects to assist students in understanding mathematics. The Cabri Jr. application can be pre-loaded on TI-83 Plus and TI-84 models of graphing calculators. Students can build geometric constructions interactively with points, lines, polygons, circles, and other basic objects. By altering geometric figures, students can visualize and extend patterns, make generalizations, and arrive at conclusions. Students in most middle and high schools are required to purchase or use classroom sets of calculators in the mathematics classroom. The school district and organizations such as Texas instruments provide professional development on these calculators for regular and special education teachers. Teachers have access to registering online for professional development, attending conferences (in person or virtually), and registering for activities to use with the Cabri Jr. application (http://cabrijr.com). All the activities show step-by-step instructions and topics are correlated to NCTM Standards for Geometry. Teachers may sign up online to receive notification of new activities and have access to archived activities that coincide with the scope of the district curriculum.

Additional technological resources designed to assist students with disabilities are applets, small Internet-based demonstrations and manipulatives. Applets provide animated and visual presentations for students, especially those with disabilities, to see patterns and characteristics of geometric objects, multiple representations, and other mathematics concepts through interaction with variables and objects. NCTM offers math applets through their illuminations series (http://illuminations.nctm.org/ActivitySearch.aspx). (Zorfass, et al, 2006)

Conclusions

This brief has presented background information about instructional strategies for students with disabilities that are linked to mathematics reform efforts exemplified by the NCTM standards and principles. Students with learning disabilities often experience difficulty bridging informal mathematics (concrete models of numbers) to formal abstract mathematics curriculum (symbols and notations of mathematical language). To improve acquisition and retention of essential mathematical skills and concepts, students need instructional interventions, such as CRA, and instructional strategies, such as those embodied in the NCTM process standards to promote mathematical understanding. With the implementation of CRA as one strategy and the process standards, this intervention can open doors to enable students with learning disabilities to learn more mathematics.

Mathematical tools—whether concrete manipulatives or virtual manipulatives—are supportive tools for learning. The use of mathematical tools shapes the way students think and build mathematical relationships and connections toward conceptual understanding (Fuson et al.1992). Selecting and accessing the appropriate tools and processes for students with disabilities is critical to their understanding mathematics.

Given the demands of new and existing federal legislation that requires much higher levels of proficiency and accountability, one of the biggest challenges is to provide effective mathematics instruction for students with disabilities. Implementing instruction that incorporates the process standards and selecting appropriate instructional tools and strategies for students with LD is one way to achieve mathematics literacy for *all*.

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