Developing Fluency with Basic Number Facts: Intervention for Students with Learning Disabilities

Katherine Garnett
Learning Disabilities Research & Practice (1992) 7:210216

ABSTRACT. The effects of math learning disabilities are widespread and often seriously problematic. Common among the diverse manifestations of math learning disabilities is lack of fluency with the basic number facts of addition, subtraction, multiplication, and division. Insights from both cognitive psychology and LD intervention research are gathered here to shape teachers' understanding of the processes by which number fact fluency develops. For many students with learning disabilities, current methods of seatwork drill are wholly inadequate and possibly even counterproductive. Discussion includes assessment guidelines and specific teaching considerations for increasing number fact fluency in students with learning disabilities.

BACKGROUND: ARITHMETIC DIFFICULTIES AND LEARNING DISABILITIES

Many children and youth with learning disabilities encounter substantial difficulty in learning arithmetic. Moreover, their math difficulties can persist and become seriously debilitating in adulthood. Investigations over the past decade provide portraits of diverse math learning problems among students classified as learning disabled (for reviews, see Fleischner & Garnett [1987] and Garnett & Fleischner [1987]). Table 1 provides an overview of these problems.

Table 1. Manifestations of math learning disabilities

Difficulty with:

- conceptual understanding (Kosc, 1974)
- counting sequences (Baroody, 1986)
- the written number symbol system (Russell & Ginsburg, 1984)
- the language of math (Nesher, 1982)
- basic number facts (Fleischner, Garnett, & Shepherd, 1982)
- procedural steps of computation (Cohn, 1971)
- application of arithmetic skills (Algozzine et al., 1987)
- problem solving (Fleischner, Nuzum, & Marzola, 1987; Montague & Bos, 1986)
- how arithmetic is taught in our schools (Nielsen 1990; Stevenson, 1987)

Some students experience difficulties across the entire spectrum of arithmetic areas; others exhibit striking weakness in only one or two. Some students develop conceptual understandings, but persist with erratic, unstable calculation procedures; others continue to have a confused and incomplete grasp of basic concepts. Studies have found that math learning disabilities are related to visual-spatial disorders, verbal deficits, use of immature strategies, weakness in developing auto-maticity, as well as to a variable rate of processing information (See, for example: Fleischner, Garnett, & Shepherd, 1982; Garnett & Fleischner, 1983; Geary, 1990; Hasselbring, Goin, & Bransford, 1987; 1988; Nesher, 1982; Strang & Rourke, 1985a; 1985b).

Arithmetic learning disabilities are diverse in their severity as well as in their manifestations. The most severe math disabilities have been linked to visual-spatial deficits, which often include difficulties with body-image and social perception, thus interfering with non-verbal aspects of communication, as well as with physical orientation in unfamiliar surroundings (Johnson, 1987). Even these most severe math disabilities can occur in otherwise average, bright, or gifted individuals. In the day-to-day, real world of adult functioning, math disability can pose greater problems than reading disability for a good number of individuals (Johnson & Blalock, 1986). Those with weak underlying math concepts require substantial help to make even the most basic practical use of math skills. Instructional support for these students requires: repeated teacher-guided work with physical objects, accurate and consistent *verbalizing* of their actions with those objects, and very explicit transfer of examples into real world applications, again with consistent and clear verbalizing from the student.

Other students, those with a sound underlying grasp of mathematical/spatial relations, may eventually be fine candidates for studying higher mathematics, despite needing remedial arithmetic in their elementary years (Steeves, 1983). Their math disability can often be restricted to inadequacy and frustration primarily within the elementary arithmetic curriculum with its heavy emphasis on computation. The computation-heavy curriculum puts stress on these students' weak memory, procedural sequencing, verbal self-monitoring, and automatization, rather than drawing on their underlying spatial strengths (Kosc, 1974).

Thus, it can be important for schools to examine the nature of a student's difficulties over time, distinguishing quite different long term instructional needs. Students with serious and persistent conceptual difficulties will need a viable functional curriculum with the goal of basic competence in practical applications and real-world problem solving. Other students actually may have a sound conceptual base for mathematics despite their confusion in talking and understanding others' talk about it, their seeming inability to learn times tables, their leaping to add when the sign indicates subtraction, and/or their frequent missequencing of the steps of long multiplication and division. In spite of this slippage of memory and attention, these students should not be assumed to be ineligible for algebra, geometry, or other branches of higher mathematics.

Clearly, some difficulties in math learning constitute a substantial disability stemming from factors within the learner. On the other hand, large numbers of children in American elementary schools do poorly in mathematics as a consequence of inadequate math teaching (Dossey, Mullis, Lindquist, & Chambers, 1988). The poor math achievement of American students in general has been attributed largely to classroom factors. These include: too little *time* spent on arithmetic, insufficient *interaction* during math practice, and inadequate*connecting* of concepts with language, with written symbols, and with practical applications. Thus, although the poor math performance of many students with learning disabilities may indeed reflect intrinsic weaknesses, these weaknesses are likely to be seriously exacerbated by poor math instruction (Cawley & Miller, 1989).

BASIC NUMBER FACTS AND STUDENTS WITH LEARNING DISABILITIES

Teachers frequently note that "not knowing basic math facts" is a common and conspicuous difficulty, an impediment to higher-level math, and a corrosive influence on the self-confidence of students with learning disabilities. Research confirms that many of these students are seriously inefficient in calculating basic number facts (Fleischner, Garnett, & Shepherd, 1982; Goldman, Pellegrino, & Mertz, 1988). For example, Fleischner and her colleagues found that 6 th-grade students with learning disabilities calculated basic addition facts no better than nondisabled 3rd graders. On timed assessments, 5 th graders with learning disabilities completed only one-third as many multiplication fact problems as their nondisabled counterparts. Similar results were obtained on addition and subtraction facts for 3rd and 4th graders. Interestingly, the students with learning disabilities were very much slower, but not significantly less accurate, than their nondisabled peers. Additionally, they demonstrated basic conceptual understanding of the basic math operations.

Thus, many students with learning disabilities establish basic understanding of the number relations involved in basic facts but continue using circuitous strategies long after their nondisabled peers have developed fluent performance. The term "fluency" highlights the notion of automaticity of highly practiced, routinized aspects of performance-a notion that has been useful in characterizing a certain stage in the development of skilled reading (Chall, 1983; Samuels, 1979; 1987). Fluency with basic number facts, like fluency in reading, implies sufficient automaticity of subskills such that attentional resources need be diverted towards them only minimally for smooth coordination within complex operations. As with swift word recognition and fluency in reading text, developing number-fact fluency normally occurs with sufficient practice over a considerable time period.

Knowing math facts is like spelling in that it is a highly visible, public aspect of performance. Everybody seems to notice when you're *not* good at it. Also, like spelling, at a certain point math facts are deemed easy, so that anyone who doesn't "just know 'em" risks significant and frequent blows to the ego: "You don't know that? That's easy !" Peers remarks can easily wound. Parents often lose patience: "After all these years, he still doesn't know his number facts!" or "She takes forever working them out, making those marks all over the page!" or "I don't understand why he touches his lips for counting up every single problem!"

Frequently, teachers respond to students' lagging performance by narrowing arithmetic periods to a relentless stream of computation worksheets, drill cards, and repetitious numberline hopping-sometimes year after year, often with little effect. There has been alarm at the degree to which American schools have constricted the mathematics taught in the elementary years. The National Council of Teachers of Mathematics (1989; 1990) and the National Academy of Sciences (1989) make a strong case for not consigning our students to perpetual (and often ineffective) computational practice. It seems important to refocus our instruction to make durable progress with number fact skill; we could be doing this job more effectively. At the same time, it is important that teachers expand, rather than diminish, the fullness of the math curriculum-even for those students with significant math learning problems. After all, writing involves much more than accurate spelling, and mathematics is more than number facts.

NUMBER FACTS: CONTRIBUTIONS FROM COGNITIVE PSYCHOLOGY

Insights from cognitive psychology can be useful in refocusing instruction, as they reveal *how* children normally proceed from primitive mental maneuvers to more proficient ones. Such knowledge can determine *what* should be practiced by children at different levels of proficiency. Numerous studies have detailed the enormous complexity of mental processes in even the simplest computation, charting children's thinking as it develops over time (Ashcraft, 1990; Baroody, 1985; Bisanz & LeFevre, 1990; Groen & Parkman, 1972; Nesher, 1986; Resnick, 1983; Siegler, 1987; 1988; Siegler & Jenkins, 1989).

For example, in the normal course of learning early addition, very young children apparently start off with the strategy of "counting all," meaning that they count out separately, each of the two addends (e.g., for 2 + 4, they say *"1, 2 . . . 1, 2, 3, 4"*), using objects or fingers; they then count them all again ("1, 2, *3, 4, 5, 6*").

With practice, children commonly evolve short-cut strategies. When carrying out one of these, the "short-cut sum" (Siegler & Jenkins, 1989), children again start from "one," accompanying the verbal sequence with fingers or objects, but continue counting with the second addend ("1, *2, 3, 4, 5, 6").* Using another early strategy, children display the two addends on their fingers or with objects, recognizing the answer apparently by sight or by feel without counting. Yet another short-cut involves starting with the first addend and "counting on" the second (e.g., they say *"2 . . . 3, 4, 5, 6"* or *"3, 4, 5, 6"*).

"Counting-on-to-the- *larger*-addend" is a notably evolved and particularly advantageous strategy (e.g., for 2 + 4, saying *"4 . . . 5, 6"* or just *"5, 6"*). Children's increasing use of this efficient strategy represents an important development in their construction of mental computation scaffolding. But it imposes several requirements not called for by the "counting all," "short-cut sum," or "finger display" strategies: it requires picking out the larger addend, irrespective of the order of the problem as presented (e.g., for 3 + 8, starting with *8*), and beginning the count not with "one" but with, or just after, the larger addend. Thus, in order to use this more mature strategy, children must first be able to: (a) disengage from the order presented by a problem, (b) easily select the larger of two numbers, and (c) readily count from *various* starting points in the number system.

Another mature strategy is "linking" one problem to a related problem (e.g., for 5 + 6, thinking *"5 + 5 = 10. so 5 + 6 = 11").* As childen employ such linkages more and more frequently, their understanding of numbers as a system of sensible, interrelated information becomes elaborated into an increasingly strong network. With sufficient practice, children gradually expand their use of the more mature strategies of "counting-on-from-the-larger-addend," and "linking," as well as their use of direct retrieval from memory. Conversely, they decrease their use of less mature means such as "counting all," "short-cut sum" and "finger display." As the steps of subroutines become increasingly automatized, children also become increasingly adroit at selecting among strategies for particular problems. (See Table 2 for a quick reference to the seven strategies just discussed.)

Table 2. Children's strategies for addition facts

Strategy	**Representative use to solve 2 + 4**
Counting all	"1, 2 . . . 1, 2, 3, 4 . . . 1, 2, 3, 4, 5, 6"
Short-cut sum	"1, 2, 3, 4, 5, 6"
Finger display	Displays 2 fingers, then 4 fingers, says "6"
Counting-on-from-the-first-addend	"2 . . . 3, 4, 5, 6" or "3, 4, 5, 6"
Counting-on-from-the-larger-addend	"4 . . . 5, 6"or "5, 6"
Linking	"2 + 2 = 4, + 2 more = 6"
Retrieval	"6"

Until quite recently, it was thought that the strategies described here were hierarchical, with less mature means giving way as more mature ones evolved. This, apparently, is not entirely accurate. For one thing, children use retrieval even in early stages of computation (e.g., for 2 + 2, simply saying *"4"*). Additionally, children generally use a mixture of strategies for a long time, even after they have evolved the most efficient means (Siegler & Jenkins, 1989). Although the proportion of problems solved by primitive and lengthy counting procedures diminishes, children normally continue for some time to use a mix of more and less efficient strategies; the shift to mature strategies is gradual. Interestingly, in the Siegler and Jenkins study, two factors were related to increased use of "counting-on-from-the-larger-addend" in children who had been employing it only intermittently: (a) their own conscious awareness of the strategy and its effect, and (b) the introduction of "challenge" problems, such as 23 + 2, which made the superiority of this strategy strikingly obvious. These findings offer insight into how teachers might promote this transition in children who seem wedded to the most elaborate counting, whereas they are actually capable of shifting to more efficient means.

In summary, cognitive psychology demonstrates that learning number facts is far more complex than just practicing them until they stick; learning them includes developing and employing a number of strategies for navigating the number system. Knowing number facts is not simple, one-step remembering; knowing them entails a sufficient assortment of associations easily retrievable from memory, a well-developed network of number relationships, easily activated counting and linking strategies, and well-practiced navigational rules for when to apply which maneuver. Indeed, this often taken-for-granted skill represents no small feat, requiring several years of frequent and varied number experiences and practice before children normally attain fluency.

Those older children and adults who "know" math facts actually do not rely exclusively on direct retrieval. Although they do retrieve many directly, they also have efficient and flexible counting strategies that act as extremely useful monitoring and back-up systems (Nesher, 1986). It is important that teachers understand this notion that well-oiled counting strategies underlie number fact calculation, providing back-up when direct memory falters, for whatever reason (e.g., fatigue, summertime disuse, fluctuations in attention or memory, etc.).

ASSESSMENT AND INSTRUCTION

Can They Count?

Several areas are useful to explore when students seem not to be gaining efficiency with basic facts. First, it can be worthwhile to survey students' most basic counting skills. Such an exploration could yield surprising answers to the following questions:

> How high can the students count?
> Can they start from different points in the sequence and continue counting?
> Can they count backwards? Starting from different starting points?
> Can they count by 10s by 2s by 5s?
> Can they count these various ways accurately, with ease and consistency?

Basic counting skills form an important substrate for arithmetic performance (Baroody, 1986; Fuson, Richards & Briars, 1982; Seron & Deloche, 1987). Counting objects and counting by rote are not the same thing. Being able to rattle off the count sequence comes first, before attaching it to physical objects in one-to-one relationship. Children also need to be adept with extended rote counting, with skip counting, counting by 10s and by 5s, counting from various starting points, as well as counting backwards (although this can be considerably more difficult than the other counting skills). When basic math performance lags significantly, it is important to fully explore these underlying counting skills-even for students entering adolescence.Strengthening students' counting skills can be key to strengthening their math performance (Charbonneau & John-Steiner, 1988; Sharp, 1971). Working on oral counting in regular, short, but rousing, sessions is a valuable start, attuning students' ears to the logic of the verbal patterns. The same patterns are evident in the sequence of written symbols, so it is also useful to develop students' skills with extended number lines and number grids, ensuring that they verbalize the count sequences along with the written equivalents. Much interaction and verbalizing is needed to develop students'
flexible and facile counting skills.

What Strategies Are They Using?

Beyond basic counting skills, it is important to investigate the strategies students employ with basic fact problems. Asking them how they solved the number problem is one useful way to explore this. But, children can be misleading informants at times, saying "I just knew it," and "No, no, I didn't count," when you clearly saw them shuffling fingers and mumbling the count sequence. So, carefully observing, as well as querying, is necessary to determine what students are actually doing. Additionally, queries convey to students the vital message that their thinking process is important and interesting, that right answers are not the be-all and the end-all. This regular interest on the part of the teacher can be a model for students' own monitoring of what they are doing-- and their developing the habit of self-monitoring is crucial to detecting those slip-ups which inevitably occur in calculation.

Table 3 offers guidelines for focusing instruction at different stages of students' strategy development.

Table 3.

For students using the earliest strategy of COUNTING ALL:

Do not leap to having them memorize number facts (even supposedly easy ones).
Model and encourage use of the "counting on" strategy, saying one number, counting on the next.
Use fingers (yours and theirs), varied sets of different objects, and number lines while practicing counting on.
Develop their habit of reading aloud each number problem.
Strengthen basic counting skills, both orally and in combination with written numerals.

. . . the early strategies of COUNTING-ON-TO-EITHER-ADDEND or SHORT-CUT SUM:

Do not leap to having them memorize number facts.
Lots of practice identifying the larger addend.
Model and encourage starting with the number name of the larger addend as the "trick" for faster counting on.
Continue using fingers, objects, and number lines for securing a robust understanding of what they are doing.
Emphasize the commutative principle (i.e., 4 + 5 = 5 + 4) in oral work, with fingers, objects, and number lines, as well as in written practice.
Encourage the habit of reading number problems aloud and verbalizing.
Strengthen basic counting skills, including extended counting, counting by twos, and counting backwards.

. . . ready to expand their use of COUNTING-ON-TO-THE-LARGER-ADDEND

Provide challenge problems with one addend larger than 10 (e.g., 26 + 3, 2 + 18, 35 + 4, 5 + 21, etc.) in oral interactive work.
Continue to highlight the commutative principle (4 + 5 = 5 + 4) in oral work, with objects, and in written practice.

. . . regularly using a mix of more mature strategies, as well as RETRIEVAL

Encourage decision strategies: "Do I just know this one?"

Press for speed (direct retrieval) with a few facts at a time, perhaps focusing at first on "ties" (2 + 2, 3 + 3, 4 + 4, etc.).
Work on small groups of problems that can be easily "linked." These include ties-plus-l (think 5 + 5 = 10, so 5 + 6 = 11) and + 9 problems (**think** 5 **+** 10 = 15 = 15, so 5 + 9 = 14).
Use objects to illustrate strategies (e.g., for 5 + 9, use a 10- frame with 9 objects inside and 5 outside the frame. Students move one object to have, "One 10 and 4 -- 14").
Intersperse practicing a small group of related facts with cumulative practice involving mixed groups.
Provoke discussions among students about the various strategies they use with different combinations.
Continue focus on the commutative principle, especially in oral work and when showing proofs using objects.
Continue their habitually reading number problems aloud and verbalizing.

A sturdy foundation for "speed" work has been established when students:

1. Easily demonstrate examples with materials and number lines;
2. Routinely use the commutative property (3 + 4 = 4 + 3; 5 x 6 = 6 x 5);
3. Answer some fact problems swiftly;
4. Show some degree of flexible thinking in reaching other answers; and
5. Routinely verbalize the problem (perhaps mumbling).

With this foundation, it is appropriate to mount a major speed campaign. Students' involvement in setting speed goals and keeping track of their progress can be crucial to sustaining their engagement (Fuchs, Bahr, & Bahr, 1990). They can chart progress for each target group of problems, as well as for mixed reviews. As students speed up sufficiently with each target group, they can blacken them out on facts charts, noting at each step how many are mastered and how many remain.

This sort of self-monitoring is useful. For one thing, it gives students a realistic sense of the dimensions of the task. Try asking a student who has been struggling unsuccessfully with learning his or her times tables, "How many times problems does school expect you to know really fast?" Many otherwise sharp youngsters will proffer with great seriousness and often a great sigh- "Thousands." Such unrealistic assessment breeds hopelessness. By whipping out a chart with the hundred multiplication facts and marking out those they already know, teachers can rekindle the hope needed to approach a challenge.

Alternative Groupings for Teaching Facts

A number of alternative teaching schemes have organized around ties, or doubles facts (i.e., 2 + 2, 3 + 3,
etc.). Direct retrieval apparently occurs much more readily with ties, with n + 0 and with n + 1 facts (Ashcraft, 1985). These alternative remedial schemes work on related combinations in clusters that highlight "linking," "thinking," or "generalization" strategies. Table 4 presents a regrouping of the teaching sequence for addition facts. Table 5 presents a regrouping of multiplication facts. These

alternate groupings follow lines first proposed by Thiele (1938), later by Stern (1965), and most recently by Thornton (1978) and Thornton and Toohey (1985).

Table 4. Alternate teaching sequence for addition facts*

[l] + 1 and + 0 Principles
Adding 1 or 0 to any number

[2] Ties 2 +2 ; 3 +3 ; 4 +4 ; 5 +5 ; 6 +6 ; 7 +7 ; 8 +8 ; 9 +9

[3] Ties +1 2 +3 ; 3 +4 ; 4 +5 ; 5 +6 ; 6 +7 ; 7 +8 ; 8 +9

[4] Ties +2 2 +4 ; 3 +5 ; 4 +6 ; 5 +7 ; 6 +8 ; 7 +9

[5] +10 Principle
from 2 + 10 through 10 + 10

[6] +9 Facts
from 2 + 9 through 9 + 9
Use the linking strategy-(*n* + 10)-l

[7] Remaining Facts

2+5	2+6	2+7	2+8
	3+6	3+7	3+8
		4+7	4+8
			5+8

*Must include major emphasis on the commutative principle (5 + 6 = 6 + 5).

Table 5. Alternate teaching sequence for multiplication facts*

[a] x 1 and x 0 Principles
Multiplying any number by 1 or by 0

[b] x 2/2 x

[c] x 5/5 x

[d] x9/9 x

[e] Perfect Squares
(1 x 1, 2 x 2. 3 x 3, 4 x 4, 5 z 5, 6 z 6, 7 z 7, 8 x 8, 9 x 9, 10 x 10)

[f] Remaining Facts

3 x 4	3 x 6	3 x 7	3 x 8

4 x 6	4 x 7	4 x 8
	6 x 7	6 x 8
		7 x 8

*Must include major emphasis on the commutative principles (5 x 6 = 6 x 5).

This emphasis appears worthwhile, perhaps especially for those children with learning disabilities who tend not to develop spontaneous strategies. Thornton's (1978) empirical work suggests that emphasizing such linking strategies may make a difference for students who are reasonably accurate, but not fluent, with number facts. Other evidence, though, suggests that several other aspects of instruction may be equally significant, in particular: interactive practice, frequent self-verbalizing, teacher modeling of thinking steps, and self-monitoring of progress (Fleischner & Garnett, 1982).

Perhaps more critical than promoting *specific* linking strategies is prodding children to reflect regularly on their thinking and, especially, to view their actions as decisions. As decision makers, students assume a purposeful, active role in monitoring their performance and, in the broadest sense, stay connected with the semantics beneath drill and practice (Brown & Campione, 1986).

Further Investigations

Future research needs to investigate the effect of Siegler and Jenkins' (1989) challenge problems as a teaching device for students who are using elaborate counting despite being ripe for shifting to more efficient means. In investigating the effects of challenge problems, many of the guidelines offered here would be useful, especially the emphasis on interactive, oral work. Regularly including challenge problems in student/teacher interactive math work could well promote the "mental math" prowess needed by so many students with learning disabilities who cling to number lines and paper-pencil routines.

Another, much broader area for future research is inquiry into what teachers are actually doing and not doing in their math efforts with students who have various degrees of math disability. It is not at all clear what use teachers make of available information concerning more and less effective math instruction. What do they do, day to day, as they engage in what they view as "math time"? How often, and how, do teachers practice math facts with their students? How do they make use of concrete objects? What sort of verbalizing do they promote? What groups of facts get practiced together and with what focus? How rich in problem-solving and real world applications is the day-to-day work? What proportion of time is spent in silent isolated seatwork? And what ideas and images do teachers and students have about what they do during math time?

These questions call for qualitative, naturalistic, or ethnographically-oriented research approaches in order to reveal mismatches between math teaching and learning. Preliminary work in this direction offers valuable examples and insight (Kandl, Miller, Malone, Wong, & Treagust, 1991). Clearly, effective math learning requires effective teaching; as we proceed towards solutions, we need to deal with both sides of this equation.

REFERENCES

Algozzine, B., O'Shea, D.J., Crews, W.B., & Stoddard, K. (1987). Analysis of mathematics competence of learning disabled adolescents. *The Journal of Special Education, 21*(2), 97-107.

Ashcraft, M.H. (1985). Is it farfetched that some of us remember our arithmetic facts? *Journal for Research in Mathematics Education, 16,* 99-105.

Ashcraft, M.H. (1990). Strategic processing in children's mental arithmetic: A review and proposal. In D.F. Bjorklund (Ed.) *Children's strategies: Contemporary views of cognitive development* (pp. 185- 211). Hillsdale, NJ: Lawrence Erlbaum.

Baroody, A.J. (1985). Mastery of basic number combinations: Internalization of relationships or facts? *Journal for Research in Math-matics Education, 16,* 83-98.

Baroody, A.J. (December 1986). Counting ability of moderately and mildly handicapped children. *Education and Training of the MentallyRetarded, 21,* 289-300.

Bisanz, J., & Lefevre, J. (1990). Strategic and nonstrategic processing in the development of mathematical cognition. In D.F. Bjorklund (Ed.) *Children's strategies: Contemporary views of cognitive development*
(pp. 215-244). Hillsdale, NJ: Lawrence Erlbaum.

Brown, A.L., & Campione, J.C. (1986). Psychological theory and the study of learning disabilities. *American Psychologist, 14*(10), 1059-1068.

Cawley, J.F., & Miller, J.H. (1989). Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities, 22*(4), 250-254, 259.

Chall, J. (1983). *Stages of reading development.* New York: McGraw-Hill.

Charbonneau, M.P., & John-Steiner, V. (1988). Patterns of experience and the language of mathematics. In R.R. Cocking & J.P. Mestre (Eds.), *Linguistic and cultural influences on learning mathmatics* (pp. 91-100). Hillsdale, NJ: Lawrence Erlbaum.

Cohn, R. (1971). Arithmetic and learning disabilities. In H.R. Myklebust (Ed.), *Progress in learning disabilities.* N.Y.: Grune & Strat-Dossey, J.A., Mullis, I.V.S., Lindquist, M.M., & Chambers, D.L. (1988). *The mathematics report card: Are we measuring up?* Princeton, NJ: Educational Testing Service.

Fleischner, J.E. & Garnett, K. (1982). Mastery of basic number facts by learning disabled students: An intervention study. Technical Report *#17. Research Institute for the Study of Learning Disabilities.* New York: Columbia University.

Fleischner, J.E., & Garnett, K. (1987). Arithmetic difficulties. In K. Kavale, S. Forness, & M. Bender (Eds.), *Handbook of learning disabilities, volume I: Dimensions and diagnosis.* (pp. 189-209). San Diego, CA: College Hill.

Fleischner, J.E., Garnett, K., & Shepherd, M.J. (1982). Proficiency in basic fact computation of learning disabled and nondisabled children. *Focus on Learning Problems in Mathematics, 4,* 47-55.

Fleischner, J.E., Nuzum, M., & Marzola, E. (1987). Devising an instructional program to teach arthimetic problem-solving skills to students with learning disabilities. *Journal of Learning Disabilities, 20,* 214-217.

Fuchs, L.S., Bahr, C.M., & Bahr, H.J. (1990). Effects of goal structures and performance contingencies on the math performance of adolescents with learning disabilities. *Journal of Learning Dis-abilities, 22*(9), 554-560.

Fuson, K.C., Richards, J., & Briars, D.J. (1982). The acquisition and elaboration of the number word sequence. In C.J. Brainerd (Ed.), *Children's logical and mathematical cognition* (pp. 33-92). New York: Springer-Verlag.

Garnett, K., & Fleischner, J.E. (1983). Automatization and basic fact performance of normal and learning disabled children. *Learning Disability Quarterly, 6,* 223-230.

Garnett, K., & Fleischner, J.E. (1987). Mathematical disabilities. *Pediatric Annals, 16*(2), 159-176.

Geary, D.C. (1990). A componential analysis of early learning deficit in mathematics. *Journal of Experimental Child Psychology, 49* (3), 363-383.

Goldman, S.R., Pellegrino, J.W., & Mertz, D.L. (1988). Extended practice of basic addition facts: Strategy changes in learning disabled students. *Cognition and Instruction, 5*(3), 223-265.

Groen, G., & Parkman, J.M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79,* 329-343.

Hasselbring, T., Goin, L., & Bransford, J. (1987). Developing automaticity. *Teaching Exceptional Children, 19* (3). 30-33.

Hasselbring, T., Goin, L., & Bransford, J. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children, 20* (6), 1-7.

Johnson, D. (1987). Nonverbal learning disabilities. *Pediatric Annals, 16,* 133-141.

Johnson, D., & Blalock, J. (1986). *Adults with Learning Disabilities: Clinical studies.* Orlando: Grune & Stratton.
Kandl, T., Miller, L., Malone, J., Wong, K., & Treagust, D. (1991). *A naturalistic study of mathematics teachers' use of diagnostic cues to enhance student understanding.* Paper present at the 12th Annual

Ethnography in Education Research Forum, University of Pennsylvania, February 1991.

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities, 7,* 165-178.

Montague, M., & Bos, C.S. (1986). Verbal math problem solving and learning disabilities: A review. *Focus on Learning Problems in Math, 8*(2), 7-21.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM. National Council of Teachers of Mathematics (1990). *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM.

Nesher, P. (1982). Levels of description in the analysis of addition and subtraction word problems. In T.P. Carpenter, J. Moser, T.A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective.* Hillsdale, NJ: Erlbaum.

Nesher, P. (1986). Learning mathematics. *American Psychologist, 4* (10), 114-122.

Nielsen, R. (Spring 1990). Anyone can learn math: New programs show how. *American Educator,* 29-34.

Resnick, L. (1983). A developmental theory of number understanding. In H.P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 109-151). N.Y.: Academic Press.

Russell, R., & Ginsburg: H. (1984). Cognitive analysis of children's mathematical difficulties. *Cognitive and Instruction, 1,* 217-244.

Samuels, S.J. (1979). Information processing abilities and reading. *Journal of Learning Disabilities, 20*(1), 18-22.

Samuels. S.J. (1987). The method of repeated readings. *The Reading Teacher, 32,* 403-408.

Seron, X., & Deloche, G. (1987). The production of counting sequences by aphasics and children: A matter of lexical processing? In G. Deloche & X. Seron (Eds.), *Mathematical disabilities: A cognitive neuropsychological perspective* (pp. 171-200). Hillsdale, NJ: Lawrence Erlbaum.

Sharp, F.A. (1971). *These kids don't count.* San Raphael, CA: Academic Therapy.

Siegler, R.S. (1987). Strategy choices in subtraction. In J. Sloboda & D. Rogers (Eds.), *Cognitive process in mathematics* (pp. 81-106). Oxford: Oxford University Press.

Siegler, R.S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: Gen-eral, 117,* 258-275.

Siegler, R.S., & Jenkins, E. (1989). *How children discover new strategies.* Hillsdale, NJ: Lawrence Erlbaum.

Steeves, K.J. (1983). Memory as a factor in the computational efficiency of dyslexic children with high abstract reasoning ability. *Annals of Dyslexia, 33,* 141-152.

Stern, C. (1965). *Structural arithmetic.* Boston: Houghton Mifflin.

Stevenson, H. (1987 Summer). The asian advantage: The case of mathematics. *American Educator,* 26-31, 47.

Strang, J.D., & Rourke, B.P. (1985a). Arithmetic disability subtypes: The neurological significance of specific arithmetical impairment in childhood. In B.P. Rourke (Ed.), *Neuropsychology of learning disabilities* (pp. 167-183). N.Y.: Guilford.

Strang, J.D., & Rourke, B.P. (1985b). Adaptive behavior of children who exhibit specific arithmetic disabilities and associated neuro-psychological abilities and deficits. In B.P. Rourke (Ed.), *Neuro-osychology of learning disabilities* (no. 302-328). N.Y.: Guilford.

Thiele, C.L. (l938). *The contribution of generalization to the learning of addition facts: Contribution to education.* NY: Teachers College, Columbia University.

Thornton, C. (1978). Emphasizing thinking strategies in basic fact instruction. *Journal for Research in Mathematics Education, 9,* 214- 227.

Thornton, C., & Toohey, M. (1985). Basic math facts: Guidelines for teaching and learning. *Learning Disabilities Focus, 1,* 44-57..